Preface

The study of matrices occupies a singular place within mathematics. It is still an area of active research, and it is used by every mathematician and by many scientists working in various specialities. Several examples illustrate its versatility:

- Scientific computing libraries began growing around matrix calculus. As a matter of fact, the discretization of partial differential operators is an endless source of linear finite-dimensional problems.
- At a discrete level, the maximum principle is related to nonnegative matrices.
- Control theory and stabilization of systems with finitely many degrees of freedom involve spectral analysis of matrices.
- The discrete Fourier transform, including the fast Fourier transform, makes use of Toeplitz matrices.
- Statistics is widely based on correlation matrices.
- The generalized inverse is involved in least-squares approximation.
- Symmetric matrices are inertia, deformation, or viscous tensors in continuum mechanics.
- Markov processes involve stochastic or bistochastic matrices.
- Graphs can be described in a useful way by square matrices.
Quantum chemistry is intimately related to matrix groups and their representations.

The case of quantum mechanics is especially interesting: Observables are Hermitian operators, their eigenvalues are energy levels. In the early years, quantum mechanics was called “mechanics of matrices,” and it has now given rise to the development of the theory of large random matrices. See [23] for a thorough account of this fashionable topic.

This text was conceived during the years 1998–2001, on the occasion of a course that I taught at the École Normale Supérieure de Lyon. As such, every result is accompanied by a detailed proof. During this course I tried to investigate all the principal mathematical aspects of matrices: algebraic, geometric, and analytic.

In some sense, this is not a specialized book. For instance, it is not as detailed as [19] concerning numerics, or as [35] on eigenvalue problems, or as [21] about Weyl-type inequalities. But it covers, at a slightly higher than basic level, all these aspects, and is therefore well suited for a graduate program. Students attracted by more advanced material will find one or two deeper results in each chapter but the first one, given with full proofs. They will also find further information in about the half of the 170 exercises. The solutions for exercises are available on the author’s site http://www.umpa.ens-lyon.fr/~serre/exercises.pdf.

This book is organized into ten chapters. The first three contain the basics of matrix theory and should be known by almost every graduate student in any mathematical field. The other parts can be read more or less independently of each other. However, exercises in a given chapter sometimes refer to the material introduced in another one.

This text was first published in French by Masson (Paris) in 2000, under the title Les Matrices: théorie et pratique. I have taken the opportunity during the translation process to correct typos and errors, to index a list of symbols, to rewrite some unclear paragraphs, and to add a modest amount of material and exercises. In particular, I added three sections, concerning alternate matrices, the singular value decomposition, and the Moore–Penrose generalized inverse. Therefore, this edition differs from the French one by about 10 percent of the contents.

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