Introduction

In many areas of finance and stochastics, significant advances have been made since this field of research was opened by Black, Scholes and Merton in 1973. The collection of contributions in *Advances in Finance and Stochastics* reflects this variety. Necessarily, a selection of topics had to be made, and we endeavoured to choose those that are currently in the focus of active research and will remain so in future. This selection spans risk management, portfolio theory and multi-asset derivatives, market imperfections, interest-rate modelling and exotic options.

Since Föllmer and Sondermann (1986) published one of the first mathematical finance papers on risk management in incomplete markets, quantitative research has developed rapidly in this area. The first three papers of this volume represent the recent developments in this area.

In the first paper on risk management, Delbaen extends the fundamental notion of a *coherent risk measure* in two directions from the original definition in Artzner et.al. (1999): the underlying probability space is now be a general probability space (and not finite) and the class of risks that are measured is extended to encompass all random variables on this space. Using methods from the theory of convex games he is able to prove the analogies of the results of Artzner et.al. (1999) in this much more general setup. But not everything carries through identically from the discrete setup: Delbaen shows that now a coherent risk measure has to be allowed to assume infinite values, representing completely unacceptable risks. The following contribution by Föllmer and Schied also treats coherent risk measures, but only as a special case of a more general class of risk measures: the convex risk measures. The authors show that convex risk measures can be represented as a supremum of expectations under different measures, corrected by a penalty function that depends on the probability measure alone. They also connect these risk measures to utility-based risk measurement. The third article on risk management is authored by Embrechts and Novak who give a survey of recent developments in the modelling and measurement of extremal events. While the first two articles are concerned with the question of a consistent *allocation* of risk capital to a given set of risks, this article gives asymptotic answers to the question of the *probability* with which this level of risk capital will be exceeded.

The part on portfolio theory opens with a paper by Werner in which he develops a multi-period extension to the CAPM, the APT and similar factor pricing models. By measuring the risk of the assets in terms of the risk of the underlying dividend streams (instead of the one-period returns), the author is able to give conditions under which exact factor pricing relationships hold. In contrast to this portfolio-selection problem, Duan and Pliska consider the pricing of *options* on multiple co-integrated assets. Apart from providing necessary conditions for cointegration of a set of assets with GARCH-stochastic volatilities, they also study the effect that cointegrating relationships under the physical measure have on the dynamics of the assets under the equilibrium pricing measure and on the dynamics of risk premia. In the following paper, Madan, Milne and Elliott study the effects that arise when several investors use different, individual factor pricing models, and these models are aggregated. While Werner took the factor structure as given in his model, Madan et.al. want to understand where economy-wide risk factors and risk-premia arise from, they shift the focus from asset-returns to identifying and explaining investor-specific risk exposures.

Market imperfections are the theme of the next three contributions. Kabanov and Stricker consider super-hedging strategies under transaction costs. They characterise the hedging-set (the set of initial endowments that allow a self-financing super-replication) of a contingent claim in a general setup with non-constant transaction costs. In the following paper, Frey and Patie address the problem of hedging options in illiquid markets. In a simulation study they show that a hedging strategy based upon a nonlinear partial differential equation that includes liquidity effects can significantly improve the performance of the hedge. In Frey and Patie's contribution illiquidity takes the form of market impact, i.e. the transactions of a large trader move prices, but he is able to trade at any time he chooses. Rogers and Zane consider a different kind of illiquidity in the third paper of this group: Here, traders are only allowed to trade at Poisson arrival times which they cannot influence. The traders' objective is a consumption/investment problem similar to Merton (1969). Rogers and Zane establish that Merton's investment rule (investing a fixed proportion of wealth in the risky asset) is still optimal, and characterize the modification of the optimal consumption process. Using an asymptotic expansion, they assess the cost of illiquidity to the investor.

The two contributions on interest-rate modelling both build upon the market-modelling approach for observed effective interest rates by Miltersen, Sandmann and Sondermann (1997). Bhar et.al. provide an estimation methodology for a short-rate model which explicitly recognizes the fact that the short term interest-rate is unobservable. Their approach aims to connect the stochastic models for the continuously compounded short rate with the observed effective, discretely compounded rates.

Schlögl analyses this connection in the other direction and shows that every market model implies a model for the continuously compounded short rate that is uniquely determined by the interpolation method used for rates maturing between tenor dates. He provides an interpolation method which preserves the Markovian properties of discrete-tenor models but allows for continuous stochastic dynamics of the short rate.

The final set of contributions has its focus on specific pricing problems that arise in the pricing of exotic options, in particular the connection between insurance and financial markets, optimal stopping, and barrier features which all affect the payoff of the option in a nonlinear way.

The connection between the markets for insurance and financial risks has been a long-standing area of interest to Dieter Sondermann. Nielsen and Sandmann analyse in their contribution one example where this connection is particularly evident: equity-linked life and pension insurance contracts. The authors give results for the existence of a fair periodic premium and provide approximate and numerical results for their magnitude.

Optimal stopping is the theme of the contributions by Schweizer; Shepp, Shiryaev and Sulem; and Peskir and Shiryaev. Schweizer analyses the optimal stopping problems posed by Bermudan options. As Bermuda options can only be exercised in a subset of the lifetime of the option, the early exercise strategies are subject to this additional restriction. Schweizer shows under which conditions the problem can be reduced to a modified American (unrestricted) optimal stopping problem, and how super-replication strategies can be derived in this setup.

Shepp, Shiryaev and Sulem consider an option that combines American early exercise, a knockout barrier and lookback-features: the barrier version of the Russian option. Here, the early exercise strategies are restricted by the knockout barrier of the option. Despite the complicated structure of the option, they are able to provide the optimal exercise strategy and the value function of this derivative.

The following contribution by Schürger contains an analysis of the distribution, moments and Laplace transforms of the suprema of several stochastic processes – a problem with immediate applications for the pricing of barrier and lookback options. Schürger gives explicit formulae for these quantities for Bessel processes as well as for strictly stable Levy processes with no positive jumps. For this he uses an elegant transformation from the maximum of a stochastic process to its first hitting time.

The final contribution again addresses the question of optimal stopping. Peskir and Shiryaev analyse the Poisson disorder problem, the problem of detecting a change in the intensity of a Poisson process. In this context they show that the smooth-pasting condition is not always valid for the optimal value function if the state vector can be discontinuous.

All authors are leading experts in their fields and we are very grateful to them for their contributions to this volume. Special thanks also go to Anne Ruston for expert advice in language questions, Catriona Byrne and Susanne Denskus from Springer and to Florian Schröder.

Through the input of all these people this book has become a fitting present to mark the occasion of Dieter Sondermann's 65^{th} birthday: a volume of up-to-date research on honour of a creative researcher and the editor of a leading journal, who has helped shape the subject of mathematical finance.

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3. Miltersen, Kristian, Klaus Sandmann and Dieter Sondermann (1997), "Closed Form Solutions for Term Structure Derivatives with Log-Normal Interest Rates", Journal of Finance 52(1), 409–430.