Preface

The purpose of this monograph is to introduce some new aspects to the theory of harmonic functions and related topics. They are a fusion of some recent developments in non-associative functional analysis, semigroups and harmonic analysis. More specifically, we study the algebraic analytic structures of the space of bounded *complex* harmonic functions on a locally compact group G and its non-commutative analogue, the space of harmonic functionals on the Fourier algebra $A(G)$. We show that they are both the ranges of contractive projections on von Neumann algebras and therefore admit Jordan algebraic structures which are usually non-associative. This provides a natural setting to apply new methods and results from non-associative analysis, semigroups and Fourier algebras. We use these devices to study, among others, the Poisson representation of bounded complex harmonic functions on G, the semigroup structures of the Poisson space and the non-associative geometric structures of the harmonic functionals.

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