To my beloved wife Maria

Preface

This book contains the first systematic exposition of the presently known theory of K-loops. Besides this, it presents some new results and many examples. Furthermore, the two most important applications are described in detail. Since about ten years the subject of K-loops has grown rapidly, so it seemed reasonable to put things in order.

There are not many books on quasigroup and loop theory. The oldest are BRUCK's [20] and BELOUSOV's [10]. PFLUGFELDER's [94] is used as a general reference for the collection of survey articles [24]. Some basic loop theory is also contained in books on projective planes, such as PICKERT's [95]. More specialized, but with very different orientation are SABININ's recent [107] and UNGAR's most recent [119] publications. Therefore, most of the material covered has not appeared in book form before.

Chapters 1–6 try to unfold the theory in a coherent and self contained way, and could be used as a text. The only prerequisite is basic algebra, in particular, group theory. With very few exceptions[1] complete proofs are given. Examples are later given in Chapters 9,11,12. However, for a course Chapter 12 can be developed as needed to enrich the theory with examples.

Chapters 7–11 are more like research notes, and only partially suitable for the classroom. Still, proofs are concise, but complete. Additional prerequisites for Chapter 9 are some linear algebra and matrix groups over ordered fields, a bit of ring theory for Chapter 11, and a basic knowledge of special relativity in Chapter 10.

While Chapters 2–7 build up almost linearly, later sections have only the following additional dependencies: Chapter 9 and Chapter 11 both use Chapter 8 for some special results. Chapter 10 builds on Chapter 9, but not on Chapters 7,8.

We have opted not to include exercises. However, some of the remarks can be used as such.

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[1] These exceptions are some results on the isotopy of Bol and K-loops, which are not used elsewhere in the book.

The appendix is meant to put the material into a historic perspective, and should be seen as a supplement to the introduction. It is definitely not the result of conclusive research on the history of the subject.

Enumeration of theorems is straightforward. Occasionally equations are numbered (i), (ii), etc. These numbers will only be referred to locally, i.e., within a section. Remarks are not enumerated, for they are not cited in the text.

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Kaufering, September 2001 Hubert Kiechle

The graph on the cover depicts the relationship of various structures discussed in the book.