Preface

The book provides a new functional-analytic approach to evolution equations by considering the abstract Cauchy problem in a scale of Banach spaces. The usual functional analytic methods for studying evolution equations are formulated within the setting of unbounded, closed operators in one Banach space. This setting is not adapted very well to the study of many pseudodifferential and differential equations because these operators are naturally not given as closed, unbounded operators in one Banach space but as continuous operators in a scale of function spaces. Thus, applications within the setting of unbounded, closed operators require a considerable amount of additional work because one has to construct suitable closed realizations of these operators. This choice of closed realizations is technically complicated even for simple applications.

The main feature of the new functional analytic approach of the book is to study the operators in scales of Banach spaces that are constructed by simple reference operators. This is a natural setting for many operators acting in scales of function spaces. The operators are only expected to respect the scale and to satisfy certain inequalities but we can avoid completely the choice of any closed realization of these operators which is of great importance in applications. We use the mapping properties of the reference operators to prove sufficient conditions for well-posedness of linear and quasilinear Cauchy problems. In the linear, time-dependent case these conditions are shown to characterize well-posedness. A similar result in the standard setting (i.e., a time-dependent generalization of the Hille-Yosida/Lumer-Phillips theorem) is still an open problem.

The generality of the new functional analytic approach of the book is demonstrated by many applications to several mathematical and physical fields. One of the most important applications is a simultaneous treatment of some parabolic and hyperbolic equations. In the standard approach this is not possible, and hyperbolic and parabolic equations have to be treated by different and incompatible methods. In particular, the approach of the book can be used for applications to (strongly) degenerate parabolic equations appearing in connection with some physical and probabilistic problems. Classical results on hyperbolic and parabolic equations are special cases of these results. A further important example of equations of that type is the parabolic Navier-Stokes equations, which degenerates hyperbolically to the Euler equation. Hence, in contrast to the standard approach, with the new methods of the book we obtain results on Navier-Stokes equations degenerating to Euler equations in some parts of the space. Further results of this book include conditions on symbols for essential selfadjointness of pseudodifferential operators and well-posedness of Schrödinger equations, linear and quasilinear evolution equations in L^q-Sobolev spaces, and spaces of continuously differentiable functions, degenerate-elliptic boundary value problems, evolutions equations on networks, and a unified approach to both types of Kadomtsev-Petviashvili equations with periodic boundary conditions.

The book contains 5 chapters. Chapter 1 provides some functional analytic methods. The abstract theory of linear evolution equations in scales of Banach spaces is developed in chapter 2 and the abstract theory of quasilinear equations in chapter 3. Applications of the abstract methods to linear equations are given in chapter 4 and to quasilinear equations in chapter 5. The abstract part of the book, i.e. chapter 1-3, is kept completely self-contained. Assuming only basic knowledge on functional analysis of bounded, linear operators in Banach spaces, all functional analytic methods needed for further reading are proved in chapter 1. Readers experienced in functional analysis may skip chapter 1, start reading directly chapter 2, and go back to chapter 1 only occasionally. Whereas all auxiliary results in the abstract part are proved in detail in the book, we cannot continue this way of presentation in chapter 4 and 5 because detailed proofs of analytic results and methods necessary for applications would exceed the limit of this book. Therefore, results of this type are formulated in a self-contained way and for proofs we give references to standard monographs treating these topics.

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