Preface

The interdisciplinary subject of random heterogeneous materials has experienced remarkable growth since the publication of the well-known monograph *Statistical Continuum Theories* by Beran (1968). Many of these advances, especially those concerning the statistical characterization of the microstructure and its effect on the physical properties of the material, have not been treated fully in any book. One of the intents of the present book is to fill this gap. This book also distinguishes itself in that it provides a unified rigorous framework to characterize the microstructures and macroscopic properties of the widely diverse types of heterogeneous materials found in nature and synthetic products. Emphasis is placed on providing foundational theoretical methods that can simultaneously yield results of practical utility.

This book treats a wide breadth of topics, but the choice of subjects naturally reflects my own interests. The sheer enormity of the field has prevented me from covering many important topics. I apologize to those colleagues, known and unknown, who may not find enough of their own work cited in the ensuing pages.

This book is intended for graduate students and researchers from various walks of scientific life, including applied mathematicians, physicists, chemists, materials scientists, engineers, geologists, and biologists. In order to reach this broad audience, I have attempted to make the book as self-contained as possible, assuming only a rudimentary knowledge of probability theory, statistical mechanics, advanced calculus, and continuum mechanics. In cases where I have fallen short in this regard, the numerous references provided should satiate the voracious appetite for knowledge of the most curious minded among us. The book contains as many proofs and derivations of key results as can be accommodated within the aforementioned constraints. All of these features and an attempt to avoid technical jargon should make the book accessible to

the nonspecialist. Indeed, it is my hope that motivated experimentalists will find this book useful.

The book is divided into two parts. Part I describes basic concepts and recent advances in quantitatively characterizing the microstructure of random heterogeneous materials. Topics covered include the statistical mechanics of many-particle systems, the canonical n-point correlation function, lattice and continuum percolation theory, local volume-fraction fluctuations, computer-simulation methods, image analyses and reconstructions of real materials, and models of microstructures.

Part II treats a wide variety of macroscopic transport, electromagnetic, mechanical, and chemical properties of heterogeneous materials and describes how they are linked to the microstructure of model and real materials. Topics covered include homogenization theory, variational principles and rigorous bounds, phase-interchange relations, exact results, effective-medium approximations, cluster expansions, contrast expansions, and cross-property relations.

A brief description of the topics covered in each chapter of the present book is given towards the end of Chapter 1, which provides the motivation for the book and an overview of its contents. It is unique among the chapters because it is purposely written in the nontechnical style of *Scientific American* in order to introduce the key ideas to the interdisciplinary audience for which the book is intended. The book is an outgrowth of a graduate course that I teach at Princeton University. The course begins with Chapter 1 and then immediately skips to Part II and covers in varying depths Chapters 13–21. I then cover most of the material contained Chapters 2-12 of Part I and subsequently return to Part II to cover Chapters 22 and 23. I follow this sequence because the property/microstructure connection (Part II) provides a motivation for the reasons why we will ultimately need to quantify the microstructure (Part I). However, this sequence certainly does not need to be followed in a course, especially if one is primarily interested in microstructural analysis. Although there is substantial cross referencing between Parts I and II, each part has been designed to be relatively independent of the other. Nonetheless, I believe that Parts I and II make a cohesive unit, and ideally, they should be read together. Those interested in further reading on the theme of Part II of this book are referred to the recent books by Cherkaev (2000) and Milton (2001), which emphasize the general theory of composites.

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The author would be grateful for reports of typographical and other errors to be sent electronically via the following webpage for the book:

http://cherrypit.princeton.edu/book.html,

where an up-to-date errata list will be maintained.

Princeton, New Jersey   Salvatore Torquato
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