

Preface

Focusing Your Attention

We have called this book *Mathematical Vistas* because we have already published a companion book *Mathematical Reflections* in the same series;¹ indeed, the two books are dedicated to the same principal purpose — to stimulate the interest of bright people in mathematics. It is not our intention in writing this book to make the earlier book a prerequisite, but it is, of course, natural that this book should contain several references to its predecessor. This is especially — but not uniquely — true of Chapters 3, 4, and 6, which may be regarded as advanced versions of the corresponding chapters in *Mathematical Reflections*.

Like its predecessor, the present work consists of nine chapters, each devoted to a lively mathematical topic, and each capable, in principle, of being read independently of the other chapters.² Thus this is not a text which — as is the intention of most standard treatments of mathematical topics — builds systematically on certain common themes as one proceeds

¹*Mathematical Reflections* — *In a Room with Many Mirrors*, Springer Undergraduate Texts in Mathematics, 1996; Second Printing 1998. We will refer to this simply as *MR*.

²There was an exception in *MR*; Chapter 9 was concerned with our thoughts on the doing and teaching of mathematics at the undergraduate level.

systematically through the book from Chapter 1 to Chapter 9. Any chapter that takes the reader's fancy may be studied at any time; and one is as likely to find in Chapter 4 a key reference to Chapter 8 as the other way round.³

Since the chapters are quite distinct from each other — though interrelated as the parts of mathematics inevitably are — each carries its own list of references.⁴ We believe it will thus be clear to the reader why there is not simply a common list of references at the end of our text, but there is a common index.

We have chosen the topics treated with several considerations in mind. We have, as we have said, always hoped to intrigue and inform the bright and curious reader; but we do not believe that all topics should be treated at the same depth and with the same thoroughness. At one end of the spectrum we have the content of Chapters 3, 4, and 6, where students may be expected to have the necessary background for a complete understanding,⁵ and where the proofs themselves constitute, in our judgment, an essential part of the appeal of the topics treated. At the other end of the spectrum we have Chapter 2, on Fermat's Last Theorem. The amazing triumph of Andrew Wiles in proving a famous result conjectured over 300 years ago is now — and forever will remain — a notable event in the history of the mathematical sciences. A book designed to highlight the attractions of mathematics should obviously tell the reader about this event, but it would be quite unrealistic to expect our readers to have the necessary technical background to be able to follow every step of the subtle argument used by Wiles and his colleagues. So we have discussed the history of FLT⁶ in some detail, and sketched the arguments used in its proof.

Another consequence of the fact that the material of this book is not sequential, but rather consists of nine related mathematical essays, is that figures are sometimes repeated. We have preferred to repeat a figure rather than oblige our readers to study a figure that may be in a part of the book far from the pages currently being read.

We have thought of Chapters 1 and 2 as appetizers, though of a very different kind. Chapter 1 (Paradoxes) is quite self-contained and quite elementary; but it does serve to show that it is not sufficient to *learn* mathematics — one must also *think* about it. Unfortunately, many students of

³Actually — and exceptionally — we *do* recommend that Chapter 4 be read before Chapter 8, because Chapter 4 provides so much of the raw material for the examples used in Chapter 8.

⁴We include, in some cases, references to the web, which most of you will readily recognize from the format.

⁵Such a background is provided by a reading of Chapters 3, 4, 6, respectively, of *MR*.

⁶As Fermat's Last Theorem is abbreviated by all mathematicians.

mathematics today seem to believe it suffices just to commit mathematical techniques and arguments to memory, and regard it as almost a breach of contract if they are asked actually to reason mathematically. So Chapter 1 is included to help the reader to appreciate the proper role of thinking in doing mathematics, at any level.

Chapter 2 (FLT) is in our book to show how remarkable and rewarding mathematics is; but, of course, as we have said, it does not claim to be a complete treatment of its subject matter. The remaining chapters, however, are comprehensive in that they give clear⁷ indications of all relevant arguments. Where we believe that the arguments or concepts are really difficult to grasp we have *starred* the material, that is, we have drawn a wavy line in the left-hand margin, with its initial and terminal points marked by a star. Sometimes an entire section has been starred. Starred material may, of course, be omitted on first reading if preferred.

Let us now describe some special features of our text. We have included a number of *BREAKS* in each chapter. These breaks consist of problems designed to enable our readers to test their understanding of the material thus far in the chapter. Answers to some of the problems appear at the end of the book. A complete set of answers to the problems in the breaks is available on request from Jean Pedersen, Department of Mathematics and Computer Science, Santa Clara University, Santa Clara, CA 95053.

Items in each chapter are numbered consecutively through the chapter. If an item in one chapter is referred to in another, then the chapter number is also given in the reference. We believe that the only conceivable confusion the reader may find in our numbering system is that n may refer either to formula (n) in the chapter or to the n th question of a given break. However, we have tried to ensure that the context always makes clear to which we are referring.

We adopt the standard abbreviations RHS and LHS for the *right-hand side* and *left-hand side*, respectively, of an equation, and the standard tombstone symbol $\square$ to mark the end of a proof.

In connection with reading proofs, we advise the reader to follow, where appropriate, one of our guidelines (in Chapter 9 of *MR*), and to look at a *particular but not special* case; that is, to replace one of the parameters appearing in the argument by a particular, but not special, numerical value. By substituting a *particular* value the reader may well make the argument easier to understand; by avoiding a *special* value, the reader does not risk failing to recognize the true nature of the argument.

⁷We hope!

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