To my mother, Irene

## Foreword

An encouraging trend in the field of artificial intelligence in the past few years has been its growing interaction with the area of operations research. AI has traditionally concentrated on problems of logical inference and satisfiability, or in other words, Boolean feasibility problems. OR, by contrast, has mainly focused on problems of linear optimization. Many significant real world tasks share aspects of both kinds of problems, and there is therefore much interest in integrating and expanding the techniques that have been developed in each field. This monograph by Dr. Joachim Paul Walser is thus particularly timely and significant. He developes new algorithms and systems for applying discrete local search techniques that were originally developed for Boolean feasibility problems to a broad class of integer linear optimization problems. He demonstrates that his general, domain-independent solver can be competitive with specialized algorithms on hard realistic problems, and can often far outperform other state-of-the-art domain-independent solvers. Throughout the monograph Dr. Walser draws connections to classic techniques from OR and AI, and demonstrates how different approaches (such as local search and linear relaxations) can be combined to solve relevant problems in integer programming.

The contributions of this monograph can be placed in two groups: First, there is the specific algorithmic work, including extending the "walksat" algorithm from Boolean to integer constraints and creating a new local search strategy for over-constrained problems. The resulting WSAT(OIP) system represents the distillation of a careful and deep search through the space of possible designs, and its elegance, and empirical success are remarkable. To take one specific example: walksat's performance is known to degrade in the presence of constraints containing many variables. However, WSAT(OIP) can efficiently handle the very long constraints (over 300 variables per constraint) that arise in many real-world domains.

Second, the book presents a series of well-chosen empirical evaluations. The different cases represent a spectrum of different kinds of problems: feasibility versus optimization, loosely-constrained versus highly constrained, and 0/1 versus integer valued variables. Some of the most significant results Dr. Walser reports are on problems of capacitated production planning. These are very large, hard industrial problems, which are (unfortunately) all too

rare in academic research. Dr. Walser demonstrates that WSAT(OIP) can find solutions that are much closer to optimal than could be found by any competing approach. The empirical evaluation shows, for the first time, that a completely general local search engine (as opposed to domain-specific local search algorithms) can efficiently find optimal or near-optimal solutions to a broad range of real-world problems, and thus complement established systematic problem-solving frameworks such as integer programming branchand-bound.

In short, this material is mandatory reading for researchers in AI who are seriously concerned with solving hard combinatorial search and optimization problems, as well as those researchers in OR who want to see the best that AI has to offer. The clarity and breadth of the presentation also makes this book an excellent choice for reading material in a graduate or advanced undergraduate seminar in either AI or OR.

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## Preface

Software to support complex planning decisions is becoming a vital factor for competitiveness, driven by the increasing availability of organizational data in modern enterprise information systems. Decision support software can reduce manufacturing costs, increase organizational efficiency, and deliver solutions to complex resource allocation problems – by building on effective optimization algorithms.

Integer optimization covers a variety of practically important optimization problems, including production planning, timetabling, VLSI circuit design, network design, logistics, or sports scheduling. The goal of integer optimization is to solve a system of constraints over many discrete variables, and to find solutions that are 'good' in terms of given optimization criteria. While fast general-purpose algorithms for solving large systems of linear inequalities over continuous variables are well-established (linear programming), integer optimization problems which include discrete decisions pose a difficult challenge to algorithmics. Yet, discrete decisions ("the truck leaves *either* today *or* tomorrow") are a critical part of most real-world planning and scheduling scenarios.

In the recent past, the field of integer and combinatorial optimization has gained momentum, and amongst the many new algorithms, *heuristics* have taken a leading role in finding near-optimal solutions to specific optimization tasks. The success of special-purpose heuristics is mainly due to their effectiveness for large practical problems – even if they come with no theoretical guarantee of optimality. The drawback of special-purpose algorithms, however, is their limited applicability. As a result, many practical optimization problems are still attacked in an ad-hoc fashion since practitioners often lack the time and expertise to research and develop effective special-purpose algorithms for the diverse optimization problems that arise.

This monograph explores a new domain-independent approach to integer optimization, which, unlike traditional strategies for integer optimization, is based on local search. It develops the central ideas and strategies of *integer local search* and describes possible combinations with classical methods, such as linear programming. In a number of case studies, it demonstrates the surprising effectiveness of the approach for a variety of realistic discrete optimization problems.

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Like traditional strategies for integer linear programming, integer local search operates on an abstract model of the problem to be solved and can thereby exploit the underlying commonalities shared by many real-world problems. As a result, solvers based on the technology described here can be combined with existing off-the-shelf modeling languages for integer programming and can be applied to many integer optimization problems without the need of code implementation. We investigate the potential of integer local search for various domains (time tabling, sports scheduling, radar surveillance, course assignment, and capacitated production planning) and compare the experimental results to state-of-the-art integer programming and constraint programming approaches.

This book is written for researchers and practitioners in the area of combinatorial optimization from artificial intelligence and operations research. Developers with an interest in the design of optimization algorithms will benefit from the detailed description of new local search strategies. Practitioners in the field can obtain insights into modeling issues and learn about the capabilities of integer local search, which often surpasses state-of-the-art IP solvers for the domains under investigation.

This book is organized as follows. Chapter 1 introduces the context in which this work is situated, integer optimization, heuristics, and local search. It provides a high-level description of the basic strategy of integer local search and the underlying representation of over-constrained integer programs. It also outlines the experimental results from the application case studies. Chapter 2 briefly introduces important general frameworks for combinatorial optimization and their terminology, i. e. integer linear programming branch-and-bound, finite domain constraint programming, and local search. It also discusses complementary search relaxations as a new characteristic to classify optimization methods.

Chapter 3 contains the technical contributions, over-constrained integer programs and presents an in-depth description of the integer local search method WSAT(OIP). It also discusses several possible combinations with linear programming and illustrates different variations of the WSAT(OIP) strategy with graphical examples. Chapter 4 describes the case study methodology. It discusses criteria of success for practical optimization methods and motivates the experimental design and the problem selection.

The remainder of the text describes the case studies, each chapter focusing on a particular problem type, and providing evidence that important criteria of success are met by integer local search. Chapter 5 describes applications in time-tabling and sports scheduling, Chapter 6 radar surveillance and course assignment, and Chapter 7 presents an application to capacitated production planning. Chapter 8 finally discusses limitations and extensions of the current methods and concludes with suggestions for future work.

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