PREFACE

These notes consist of two parts:

1) Selected Topics in Geometry, New York University 1946, Notes by Peter Lax.

2) Lectures on Differential Geometry in the Large, Stanford University 1956, Notes by J.W. Gray.

They are reproduced here with no essential change.

Heinz Hopf was a mathematician who recognized important mathematical ideas and new mathematical phenomena through special cases. In the simplest background the central idea or the difficulty of a problem usually becomes crystal clear. Doing geometry in this fashion is a joy. Hopf's great insight allows this approach to lead to serious mathematics, for most of the topics in these notes have become the starting-points of important further developments. I will try to mention a few.

It is clear from these notes that Hopf laid the emphasis on polyhedral differential geometry. Most of the results in smooth differential geometry have polyhedral counterparts, whose understanding is both important and challenging. Among recent works I wish to mention those of Robert Connelly on rigidity, which is very much in the spirit of these notes (cf. R. Connelly, Conjectures and open questions in rigidity, Proceedings of International Congress of Mathematicians, Helsinki 1978, vol. 1, 407-414).

A theory of area and volume of rectilinear polyhedra based on decompositions originated with Bolyai and Gauss. Gauss realized the delicacy of the problem for volumes, and Hilbert proposed in his famous "Mathematical Problems" that of "constructing two tetrahedra of equal bases and equal altitudes which can in no way be split into congruent tetrahedra..." (Problem no. 3). This was immediately solved by Max Dehn whose results, with some modifications, are presented in Part 1, Chapter IV of these notes. This work has been further pursued and treated by algebraic methods. For the modern developments I refer to C.H. Sah, Hilbert's third problem: Scissors congruence (Research Notes in Mathematics 33, Pitman, San Francisco 1979).

The main content of Part 2 consists of the study of Weingarten surfaces in the three-dimensional Euclidean space, particularly those for which the mean curvature or the Gaussian curvature is a constant. Important progress was recently made by Wu-Yi Hsiang, as he constructed many examples of hypersurfaces of constant mean curvature in the Euclidean space which are not hyperspheres; cf. Wu-Yi Hsiang, Generalized rotation hypersurfaces of constant mean curvature in the Euclidean spaces I (J. Differential Geometry 17 (1982), 337-356), and his other papers. But the simplest question as to whether there exists an immersed torus in the three-dimensional Euclidean space with constant mean curvature remains unanswered (the "soap bubble" problem).

Hopf's mathematical exposition is a model of precision and clarity. His style is recognizable in these notes.

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March 1983