Preface

This book presents complex analysis of several variables from the point of view of the Cauchy-Riemann equations and integral representations. A more detailed description of our methods and main results can be found in the introduction. Here we only make some remarks on our aims and on the required background knowledge.

Integral representation methods serve a twofold purpose: 1° they yield regularity results not easily obtained by other methods and 2°, along the way, they lead to a fairly simple development of parts of the classical theory of several complex variables. We try to reach both aims. Thus, the first three to four chapters, if complemented by an elementary chapter on holomorphic functions, can be used by a lecturer as an introductory course to complex analysis. They contain standard applications of the Bochner-Martinelli-Koppelman integral representation, a complete presentation of Cauchy-Fantappiè forms giving also the numerical constants of the theory, and a direct study of the Cauchy-Riemann complex on strictly pseudoconvex domains leading, among other things, to a rather elementary solution of Levi's problem in complex number space $\mathbb{C}^n$. Chapter IV carries the theory from domains in $\mathbb{C}^n$ to strictly pseudoconvex subdomains of arbitrary — not necessarily Stein — manifolds. We develop this theory taking as a model classical Hodge theory on compact Riemannian manifolds; the relation between a parametrix for the real Laplacian and the generalised Bochner-Martinelli-Koppelman formula is crucial for the success of the method. In Chapter V we describe the Neumann problem for the Cauchy-Riemann complex and prove, in particular, the fundamental density theorems due to Friedrichs and Hörmander. An analysis of this problem in our context leading to the main technical results of our work is given in Chapters VI and VIII, whereas Chapter VII develops the necessary machinery of integral estimates and function spaces. The book ends with applications of these technical results to complex analysis: Mergelyan's and Gleason's problem on complex manifolds, in the framework of Hölder spaces.

Prerequisites for reading this book are some acquaintance with the elementary theory of functions of several complex variables and a good knowledge of classical analysis, in particular of distributions and integration theory. The basic notions of analysis on manifolds are essential — even for domains in $\mathbb{C}^n$. There are very few instances where we rely more heavily on the theory of Stein spaces — mostly when we study strictly pseudoconvex domains in general manifolds. If one concentrates on subdomains of Stein manifolds, one can neglect these arguments.

As can be guessed from the above, we use and present many ideas going back to different mathematicians; we have tried to describe the historical development to the best of our knowledge but have probably failed at many instances. For this we apologize.

Particularly useful for us have been the textbooks by M. Range, G. de Rham and Ch. Laurent-Thiébaut; Chapters I and III follow their presentation at many places. Chapter V is based on Hörmander's work, in Chapter VII we use Krantz' paper [Kra 76], whereas the main results of Chapters IV and VI and many results of ChapterVIII are due to Lieb and Range (and to Michel in the analogous pseudoconcave case). (The original solution of the $\bar{\partial}$-Neumann problem is due to Kohn.) The systematic use of local integral formulae and its combination with the theory of compact operators, which allows to pass from $\mathbb{C}^n$ to arbitrary manifolds, can be retraced to Kerzman, Henkin and Range; Grauert's "bump method" plays a decisive role in this context, and the study of arbitrary — i. e. non-Stein — manifolds requires some classical tools which go back to Remmert, Cartan and K. Stein. Finally, the powerful analytical methods developed by E. M. Stein have been an essential help in estimating the kernels which we construct.

Our cooperation has been supported by the SFB 256 of the Deutsche Forschungsgemeinschaft, by the university of Bonn and the Université du Littoral – Côte d'Opale, and by the European network ANACOGA. We are very grateful to these institutions. Our particular thanks go to Dipl. Math. C. Wallat, Dipl. Math. Ch. Lampert and Dr. T. Hefer for competently establishing the TEX-file, and, moreover, suggesting many improvements and correcting many a mistake in the different versions of our text. Prof. K. Diederich has kindly invited us to publish our work in his "Aspects of Mathematics" series; his comments on our work have been particularly useful. Finally, Ms. U. Schmickler-Hirzebruch from Vieweg Verlag has patiently encouraged us to continue — and to come to an end.

The help of all these institutions, friends, and colleagues has been invaluable.

The first named author was a student of H. Grauert. It is a pleasure and an honour to dedicate this work to H. Grauert who, over more than 40 years, has immensely contributed to the development of complex analysis.

Bonn and Calais, January 2002 Ingo Lieb and Joachim Michel