1.3 Heat exchangers

When energy, as heat, has to be transferred from one stream of fluid to another both fluids are directed through an apparatus known as a heat exchanger. The two streams are separated by a barrier, normally the wall of a tube or pipe, through which heat is transferred from the fluid at the higher temperature to the colder one. Calculations involving heat exchangers use the equations derived in section 1.2 for overall heat transfer. In addition to these relationships, the energy balances of the first law of thermodynamics link the heat transferred with the enthalpy changes and therefore the temperature changes in both the fluids.

Heat exchangers exist in many different forms, and can normally be differentiated by the flow regimes of the two fluids. These different types will be discussed in the first part of this section. This will be followed by a section on the equations used in heat exchanger design. These equations can be formulated in a favourable manner using dimensionless groups. The calculation of countercurrent, cocurrent and cross current exchangers will then be explained. The final section contains information on combinations of these three basic flow regimes which are used in practice.

The calculation, design and application of heat exchangers is covered comprehensively in other books, in particular the publications from H. Hausen [1.7], H. Martin [1.8] as well as W. Roetzel [1.9] should be noted. The following sections serve only as an introduction to this extensive area of study, and particular emphasis has been placed on the thermal engineering calculation methods.

1.3.1 Types of heat exchanger and flow configurations

One of the simplest designs for a heat exchanger is the *double pipe heat exchanger* which is schematically illustrated in Fig. 1.19. It consists of two concentric tubes, where fluid 1 flows through the inner pipe and fluid 2 flows in the annular space between the two tubes. Two different flow regimes are possible, either countercurrent where the two fluids flow in opposite directions, Fig. 1.19a, or cocurrent as in Fig. 1.19b.

Fig. 1.19 also shows the cross-sectional mean values of the fluid temperatures ϑ_1 and ϑ_2 over the whole length of the heat exchanger. The entry temperatures are indicated by one dash, and the exit temperature by two dashes. At every cross-section $\vartheta_1 > \vartheta_2$, when fluid 1 is the hotter of the two. In countercurrent flow the two fluids leave the tube at opposite ends, and so the exit temperature of the hot fluid can be lower than the exit temperature of the colder fluid ($\vartheta_1'' < \vartheta_2''$), because only the conditions $\vartheta_1'' > \vartheta_2'$ and $\vartheta_1' > \vartheta_2''$ must be met. A marked cooling of fluid 1 or a considerable temperature rise in fluid 2 is not possible with cocurrent flow. In this case the exit temperatures of both fluids occur at the same end of the

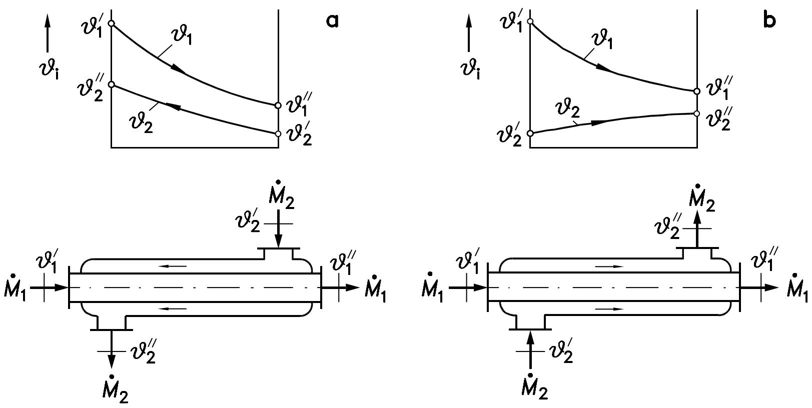

Fig. 1.19: Fluid temperatures ϑ_1 and ϑ_2 in a double-pipe heat exchanger. **a** countercurrent flow, **b** cocurrent flow

exchanger and so $\vartheta_1'' > \vartheta_2''$ is always the case, no matter how long the exchanger is. This is the first indication that countercurrent flow is superior to cocurrent flow: not all heat transfer tasks carried out in countercurrent flow can be realised in cocurrent flow. In addition to this fact, it will be shown in section 1.3.3, that for the transfer of the same heat flow, a countercurrent heat exchanger always has a smaller area than a cocurrent exchanger, assuming of course, that the both flow regimes are suitable to fulfill the task. Therefore, cocurrent flow is seldom used in practice.

In practical applications the *shell-and-tube heat exchanger*, as shown in Fig. 1.20 is the most commonly used design. One of the fluids flows in the many parallel tubes which make up a tube bundle. The tube bundle is surrounded

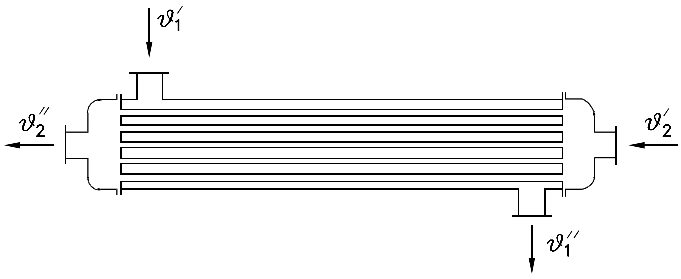

Fig. 1.20: Shell-and-tube heat exchanger (schematic)

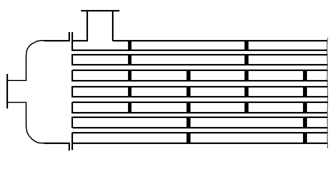

Fig. 1.21: Shell-and-tube heat exchanger with baffles

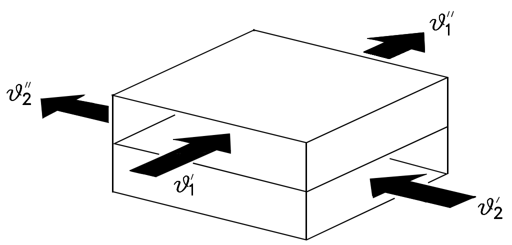

Fig. 1.22: Scheme of a plate exchanger with crossflow

by a shell. The second fluid flows around the outside of the tubes within this shell. Countercurrent flow can be realised here except at the ends at of the heat exchanger where the shell side fluid enters or leaves the exchanger. The addition of baffles, as in Fig. 1.21, forces the shell side fluid to flow perpendicular to the tube bundle, which leads to higher heat transfer coefficients than those found in flow along the tubes. In the sections between the baffles the fluid is neither in counter or cocurrent flow but in *crossflow*.

Pure crossflow is found in *flat plate heat exchangers*, as indicated by Fig. 1.22. The temperatures of both fluids also change perpendicular to the flow direction. This is schematically shown in Fig. 1.23. Each fluid element that flows in a crossflow heat exchanger experiences its own temperature change, from the entry temperature ϑ_i' which is the same for all particles to its individual exit temperature. Crossflow is often applied in a shell-and-tube heat exchanger when one of the fluids is gaseous. The gas flows around the rows of tubes crosswise to the tube axis. The other fluid, normally a liquid, flows inside the tubes. The addition of

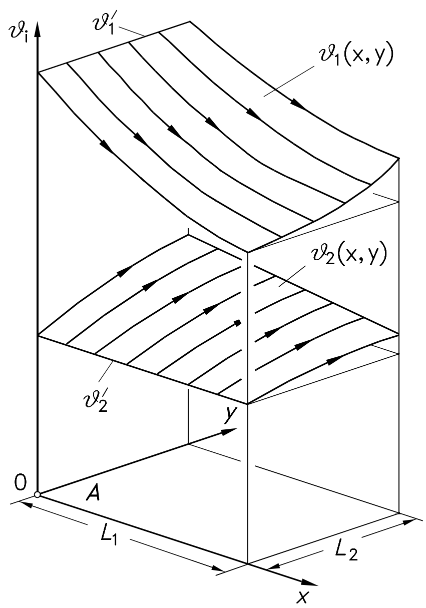

Fig. 1.23: Fluid temperatures $\vartheta_1 = \vartheta_1(x,y)$ and $\vartheta_2 = \vartheta_2(x,y)$ in crossflow

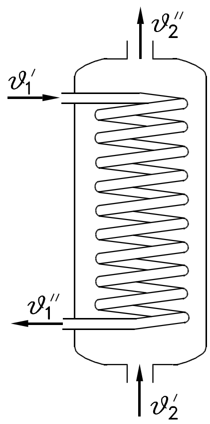

Fig. 1.24: Coiled tube heat exchanger (schematic)

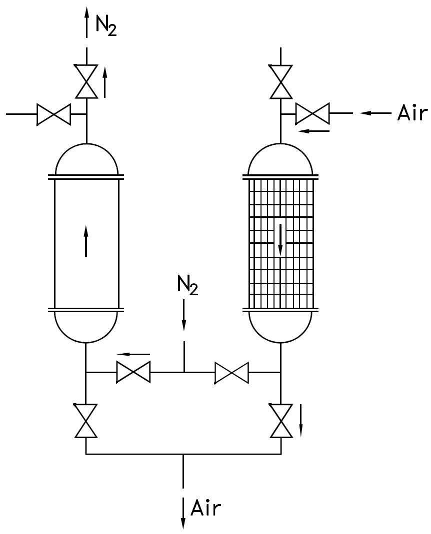

Fig. 1.25: Regenerators for the periodic heat transfer between the gases, air and nitrogen (schematic)

fins to the outer tube walls, cf. 1.2.3 and 2.2.3, increases the area available for heat transfer on the gas side, thereby compensating for the lower heat transfer coefficient.

Fig. 1.24 shows a particularly simple heat exchanger design, a *coiled tube* inside a vessel, for example a boiler. One fluid flows through the tube, the other one is in the vessel and can either flow through the vessel or stay there while it is being heated up or cooled down. The vessel is usually equipped with a stirrer that mixes the fluid, improving the heat transfer to the coiled tube.

There are also numerous other special designs for heat exchangers which will not be discussed here. It is possible to combine the three basic flow regimes of countercurrent, cocurrent and crossflow in a number of different ways, which leads to complex calculation procedures.

The heat exchangers dealt with so far have had two fluids flowing steadily through the apparatus at the same time. They are always separated by a wall through which heat flows from the hotter to the colder fluid. These types of heat exchangers are also known as *recuperators*, which are different from *regenerators*. They contain a packing material, for example a lattice of bricks with channels for the gas or a packed bed of stone or metal strips, that will allow gases to pass through it. The gases flow alternately through the regenerator. The *hot* gas transfers heat to the packing material, where it is stored as internal energy. Then the *cold* gas flows through the regenerator, removes heat from the packing and leaves at a higher temperature. Continuous operation requires at least two regenerators, so that one gas can be heated whilst the other one is being cooled, Fig. 1.25. Each of the regenerators will be periodically heated and cooled by switching the gas flows around. This produces a periodic change in the exit temperatures of the gases.

Regenerators are used as air preheaters in blast furnaces and as heat exchangers in low temperature gas liquefaction plants. A special design, the Ljungström preheater, equipped with a rotating packing material serves as a preheater for air in firing equipment and gas turbine plants.The warm gas in this case is the exhaust gas from combustion which should be cooled as much as possible for energy recovery.

The regenerator theory was mainly developed by H. Hausen [1.10]. As it includes a number of complicated calculations of processes that are time dependent no further study of the theory will be made here. The summary by H. Hausen [1.7] and the VDI-Wärmeatlas [1.11] are suggested for further study on this topic.

1.3.2 General design equations. Dimensionless groups

Fig. 1.26 is a scheme for a heat exchanger. The temperatures of the two fluids are denoted by ϑ_1 and ϑ_2, as in section 1.3.1, and it will be assumed that $\vartheta_1 > \vartheta_2$. Heat will therefore be transferred from fluid 1 to fluid 2. Entry temperatures are indicated by one dash, exit temperatures by two dashes.

The first law of thermodynamics is applied to for both fluids. The heat transferred causes an enthalpy increase in the cold fluid 2 and a decrease in the warm fluid 1. This gives

$$\dot{Q} = \dot{M}_1(h_1' - h_1'') = \dot{M}_2(h_2'' - h_2') \ , \tag{1.93}$$

where $\dot{M}_i$ is the mass flow rate of fluid i. The specific enthalpies are calculated

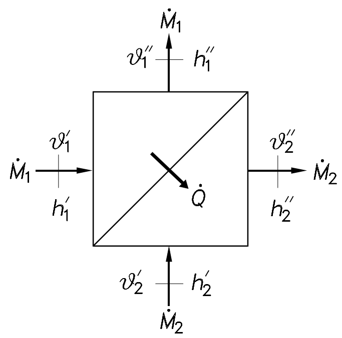

Fig. 1.26: Heat exchanger scheme, with the mass flow rate $\dot{M}_i$, entry temperatures ϑ_i', exit temperatures ϑ_i'', entry enthalpy h_i' and exit enthalpy h_i'' of both fluids $(i = 1, 2)$

at the entry and exit temperatures ϑ_i' and ϑ_i'' respectively. These temperatures are averaged over the relevant tube cross section, and can be determined using the explanation in section 1.1.3 for calculating adiabatic mixing temperatures. Equation (1.93) is only valid for heat exchangers which are *adiabatic* with respect to their environment, and this will always be assumed to be the case.

The two fluids flow through the heat exchanger without undergoing a phase change, i.e. they do not boil or condense. The small change in specific enthalpy with pressure is neglected. Therefore only the temperature dependence is important, and with

$$\overline{c}_{pi} := \frac{h_i' - h_i''}{\vartheta_i' - \vartheta_i''} \quad , \qquad i = 1, 2 \tag{1.94}$$

the mean specific heat capacity between ϑ_i' and ϑ_i'' it follows from (1.93) that

$$\dot{Q} = \dot{M}_1 \overline{c}_{p1} (\vartheta_1' - \vartheta_1'') = \dot{M}_2 \overline{c}_{p2} (\vartheta_2'' - \vartheta_2') \ .$$

As an abbreviation the *heat capacity flow rate* is introduced by

$$\dot{W}_i := \dot{M}_i \overline{c}_{pi} \quad , \qquad i = 1, 2 \tag{1.95}$$

which then gives

$$\dot{Q} = \dot{W}_1 (\vartheta_1' - \vartheta_1'') = \dot{W}_2 (\vartheta_2'' - \vartheta_2') \ . \tag{1.96}$$

The temperature changes in both fluids are linked to each other due to the first law of thermodynamics. They are related inversely to the ratio of the heat capacity flow rates.

The heat flow $\dot{Q}$ is transferred from fluid 1 to fluid 2 because of the temperature difference $\vartheta_1 - \vartheta_2$ inside the heat exchanger. This means that the heat flow $\dot{Q}$ has to overcome the overall resistance to heat transfer $1/kA$ according to section 1.2.1. The quantity kA will from now on be called the *transfer capability* of the heat exchanger, and is a characteristic quantity of the apparatus. It is calculated using (1.72) from the transfer resistances in the fluids and the resistance to conduction in the wall between them. The value for kA is usually taken to be an apparatus constant, where the overall heat transfer coefficient k is assumed to have the same value throughout the heat exchanger. However this may not always happen, the fluid heat transfer coefficient can change due to the temperature dependence of some of the fluid properties or by a variation in the flow conditions. In cases such as these, k and kA must be calculated for various points in the heat exchanger and a suitable mean value can be found, cf. W. Roetzel and B. Spang [1.12], to represent the characteristic transfer capability kA of the heat exchanger.

Before beginning calculations for heat exchanger design, it is useful to get an overview of the quantities which have an effect on them. Then the number of these quantities will be reduced by the introduction of dimensionless groups. Finally the relevant relationships for the design will be determined. Fig. 1.27 contains the seven quantities that influence the design of a heat exchanger. The effectiveness of the heat exchanger is characterised by its transfer capability kA,

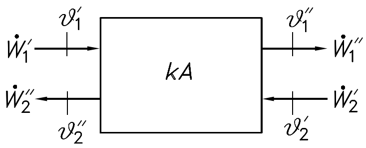

Fig. 1.27: Heat exchanger with the seven quantities which affect its design

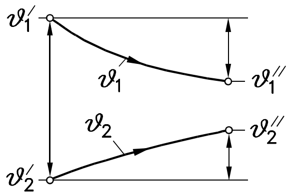

Fig. 1.28: The three decisive temperature differences (arrows) in a heat exchanger

the two fluid flows by their heat capacity flow rates $\dot W_i$, entry temperatures ϑ_i' and exit temperatures ϑ_i''. As the temperature level is not important only the three temperature differences $(\vartheta_1' - \vartheta_1'')$, $(\vartheta_2'' - \vartheta_2')$ and $(\vartheta_1' - \vartheta_2')$, as shown in Fig. 1.28, are of influence. This reduces the number of quantities that have any effect by one so that six quantities remain:

$$kA,\ (\vartheta_1' - \vartheta_1''),\ \dot W_1,\ (\vartheta_2'' - \vartheta_2'),\ \dot W_2 \quad \text{and} \quad (\vartheta_1' - \vartheta_2') \ .$$

These belong to only two types of quantity either temperature (unit K) or heat capacity flow rate (units W/K). According to section 1.1.4, that leaves four (= 6 − 2) characteristic quantities to be defined. These are the dimensionless temperature changes in both fluids

$$\varepsilon_1 := \frac{\vartheta_1' - \vartheta_1''}{\vartheta_1' - \vartheta_2'} \quad \text{and} \quad \varepsilon_2 := \frac{\vartheta_2'' - \vartheta_2'}{\vartheta_1' - \vartheta_2'} \ , \tag{1.97}$$

see Fig. 1.29, and the ratios

$$N_1 := \frac{kA}{\dot W_1} \quad \text{and} \quad N_2 := \frac{kA}{\dot W_2} \ . \tag{1.98}$$

These are also known as the Number of Transfer Units or *NTU* for short. We suggest N_i be characterised as the dimensionless transfer capability of the heat exchanger. Instead of N_2 the ratio of the two heat capacity flow rates

$$C_1 := \frac{\dot W_1}{\dot W_2} = \frac{N_2}{N_1} \tag{1.99}$$

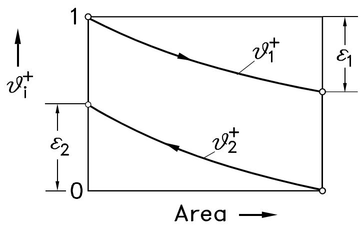

Fig. 1.29: Plot of the dimensionless fluid temperatures $\vartheta_i^+ = (\vartheta_i - \vartheta_2')\,/\,(\vartheta_1' - \vartheta_2')$ over the area and illustration of ε_1 and ε_2 according to (1.97)

or its inverse

$$C_2 := \frac{\dot{W}_2}{\dot{W}_1} = \frac{1}{C_1} \tag{1.100}$$

is often used.

The four groups in (1.97) and (1.98), are not independent of each other, because applying the first law of thermodynamics gives

$$\frac{\varepsilon_1}{N_1} = \frac{\varepsilon_2}{N_2} \quad \text{or} \quad \varepsilon_2 = C_1\varepsilon_1 \ . \tag{1.101}$$

The relationship which exists between the three remaining characteristic quantities

$$F(\varepsilon_1, N_1, N_2) = 0 \quad \text{or} \quad F(\varepsilon_1, N_1, C_1) = 0 \tag{1.102}$$

is the *operating characteristic* of the heat exchanger. It depends on the flow configuration and is found from the temperature pattern of both fluids, that will be discussed in detail in the following sections.

Heat exchanger design mainly consists of two tasks:

1. Calculating the heat flow transferred in a given heat exchanger.
2. Design of a heat exchanger for a prescribed performance.

In the first case $(\vartheta_1' - \vartheta_2')$, $\dot{W}_1$, $\dot{W}_2$ and kA will all be given. The temperature changes in both fluids have to be found so that $\dot{Q}$, the heat flow transferred, can be determined from (1.96). As the characteristic numbers, N_1 and N_2 or N_1 and C_1 are given this problem can be solved immediately, if the operating characteristic in (1.102) can be explicitly resolved for ε_1:

$$\varepsilon_1 = \varepsilon_1(N_1, C_1) \ .$$

The dimensionless temperature change ε_2 of the other fluid follows from (1.101).

In the calculations for the design of a heat exchanger kA has to be found. Either the temperature changes in both fluids or the two values for the heat capacity flow and the temperature change in one of the fluids must be known, in order to determine kA. An operating characteristic which can be explicitly solved for N_1 or N_2 is desired:

$$N_1 = N_1(\varepsilon_1, C_1) \ .$$

This gives for the transfer capability

$$kA = N_1\dot{W}_1 = N_2\dot{W}_2 \ .$$

In Fig. 1.30 an operating characteristic for a heat exchanger with given flow configuration is shown. The solutions to both tasks, heat transfer calculation and design calculation are indicated. In many cases the explicit solution of the operating characteristic for ε_1 and N_1 is not possible, even if an analytical expression is available. If this arises a diagram similar to Fig. 1.30 should be used. Further details are given in section 1.3.5.

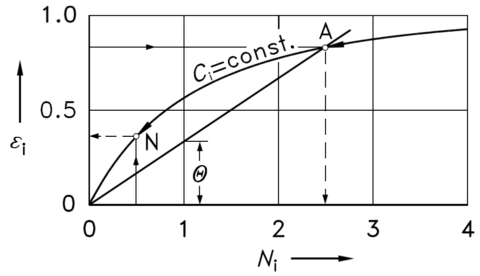

Fig. 1.30: Schematic representation of the operating characteristic for a heat exchanger with C_i = const. N is the assumed operating point for the heat transfer calculations: $\varepsilon_i = \varepsilon_i\,(N_i, C_i)$, A is the assumed operating point for the design: $N_i = N_i\,(\varepsilon_i, C_i)$. The determination of the mean temperature difference Θ for point A is also shown.

When the heat capacity flow rate $\dot{W}_i$ was introduced in (1.95) boiling and condensing fluids were not considered. At constant pressure a pure substance which is boiling or condensing does not undergo a change in temperature, but $c_{pi} \to \infty$. This leads to $\varepsilon_i = 0$, whilst $\dot{W}_i \to \infty$ resulting in $N_i = 0$ and $C_i \to \infty$. This simplifies the calculations for the heat exchanger, as the operating characteristic is now a relationship between only two rather than three quantities, namely ε and N of the other fluid, which is neither boiling nor condensing.

In heat exchanger calculations another quantity alongside those already introduced is often used, namely the *mean temperature difference* $\Delta\vartheta_{\rm m}$. This is found by integrating the local temperature difference $(\vartheta_1 - \vartheta_2)$ between the two fluids over the whole transfer area.

$$\Delta\vartheta_{\rm m} = \frac{1}{A}\int\limits_{(A)} (\vartheta_1 - \vartheta_2)\;{\rm d}A \ . \tag{1.103}$$

In analogy to (1.71) the heat flow transferred is

$$\dot{Q} = kA\Delta\vartheta_{\rm m} \ . \tag{1.104}$$

This equation can only strictly be used if the heat transfer coefficient k is the same at each point on A. If this is not true then (1.104) can be considered to be a definition for a mean value of k.

The introduction of $\Delta\vartheta_{\rm m}$ in conjunction with (1.104), gives a relationship between the heat flow transferred and the transfer capability kA, and therefore with the area A of the heat exchanger. This produces the following equations

$$\dot{Q} = kA\Delta\vartheta_{\rm m} = \dot{W}_1(\vartheta_1' - \vartheta_1'') = \dot{W}_2(\vartheta_2'' - \vartheta_2') \ .$$

With the dimensionless mean temperature difference

$$\Theta = \frac{\Delta\vartheta_{\rm m}}{\vartheta_1' - \vartheta_2'} \tag{1.105}$$

the following relationship between the dimensionless groups is found:

$$\Theta = \frac{\varepsilon_1}{N_1} = \frac{\varepsilon_2}{N_2} \ . \tag{1.106}$$

The mean temperature $\Delta\vartheta_{\mathrm{m}}$ and its associated dimensionless quantity Θ can be calculated using the dimensionless numbers that have already been discussed. The introduction of the mean temperature difference does not provide any information that cannot be found from the operating characteristic. This is also illustrated in Fig. 1.30, where Θ is the gradient of the straight line that joins the operating point and the origin of the graph.

1.3.3 Countercurrent and cocurrent heat exchangers

The operating characteristic $F(\varepsilon_i, N_i, C_i) = 0$, for a countercurrent heat exchanger is found by analysing the temperature distribution in both fluids. The results can easily be transferred for use with the practically less important case of a cocurrent exchanger.

We will consider the temperature changes, shown in Fig. 1.31, in a countercurrent heat exchanger. The temperatures ϑ_1 and ϑ_2 depend on the z coordinate in the direction of flow of fluid 1. By applying the first law to a section of length dz the rate of heat transfer, $\mathrm{d}\dot{Q}$, from fluid 1 to fluid 2 through the surface element dA is found to be

$$\mathrm{d}\dot{Q} = -\dot{M}_1 c_{p1}\,\mathrm{d}\vartheta_1 = -\dot{W}_1\,\mathrm{d}\vartheta_1 \tag{1.107}$$

and

$$\mathrm{d}\dot{Q} = -\dot{M}_2 c_{p2}\,\mathrm{d}\vartheta_2 = -\dot{W}_2\,\mathrm{d}\vartheta_2\ . \tag{1.108}$$

Now $\mathrm{d}\dot{Q}$ is eliminated by using the equation for overall heat transfer

$$\mathrm{d}\dot{Q} = k\,(\vartheta_1 - \vartheta_2)\,\mathrm{d}A = kA\,(\vartheta_1 - \vartheta_2)\,\frac{\mathrm{d}z}{L} \tag{1.109}$$

from (1.107) and (1.108) giving

$$\mathrm{d}\vartheta_1 = -(\vartheta_1 - \vartheta_2)\,\frac{kA}{\dot{W}_1}\,\frac{\mathrm{d}z}{L} = -(\vartheta_1 - \vartheta_2)\,N_1\,\frac{\mathrm{d}z}{L} \tag{1.110}$$

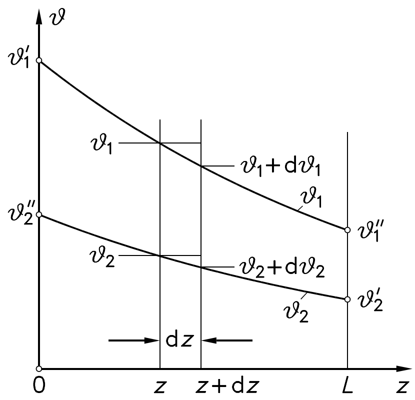

Fig. 1.31: Temperature pattern in a countercurrent heat exchanger

Table 1.4: Equations for the calculation of the normalised temperature variation ε_i , the dimensionless transfer capability N_i and the mean temperature difference Θ in counter and cocurrent heat exchangers

Flow regime	$\varepsilon_i = \varepsilon_i\,(N_i,\,C_i)$	$N_i = N_i\,(\varepsilon_i, C_i)$	$\Theta = \Theta\,(\varepsilon_1, \varepsilon_2)$
counter current $C_i \neq 1$ $i = 1, 2$	$\varepsilon_i = \dfrac{1 - \exp\left[(C_i - 1)\,N_i\right]}{1 - C_i \exp\left[(C_i - 1)\,N_i\right]}$	$N_i = \dfrac{1}{1 - C_i} \ln \dfrac{1 - C_i \varepsilon_i}{1 - \varepsilon_i}$	$\Theta = \dfrac{\varepsilon_1 - \varepsilon_2}{\ln \dfrac{1 - \varepsilon_2}{1 - \varepsilon_1}}$
$C = 1$	$\varepsilon = \dfrac{N}{1 + N}$	$N = \dfrac{\varepsilon}{1 - \varepsilon}$	$\Theta = 1 - \varepsilon$
co-current $i = 1, 2$	$\varepsilon_i = \dfrac{1 - \exp\left[-\,(1 + C_i)\,N_i\right]}{1 + C_i}$	$N_i = -\dfrac{\ln\left[1 - \varepsilon_i\,(1 + C_i)\right]}{1 + C_i}$	$\Theta = \dfrac{-\,(\varepsilon_1 + \varepsilon_2)}{\ln\left[1 - (\varepsilon_1 + \varepsilon_2)\right]}$

Meaning of the characteristic numbers: $\varepsilon_1 = \dfrac{\vartheta_1' - \vartheta_1''}{\vartheta_1' - \vartheta_2'}$, $\varepsilon_2 = \dfrac{\vartheta_2'' - \vartheta_2'}{\vartheta_1' - \vartheta_2'}$

$N_i = kA/\dot{W}_i$, $\Theta = \dfrac{\Delta\vartheta_\mathrm{m}}{\vartheta_1' - \vartheta_2'} = \dfrac{\varepsilon_i}{N_i}$, $C_1 = \dfrac{\dot{W}_1}{\dot{W}_2} = \dfrac{\varepsilon_2}{\varepsilon_1} = \dfrac{N_2}{N_1}$, $C_2 = \dfrac{1}{C_1}$

and

$$\mathrm{d}\vartheta_2 = -(\vartheta_1 - \vartheta_2)\,\frac{kA}{\dot{W}_2}\,\frac{\mathrm{d}z}{L} = -(\vartheta_1 - \vartheta_2)\,N_2\,\frac{\mathrm{d}z}{L} \tag{1.111}$$

for the temperature changes in both fluids.

The temperatures $\vartheta_1 = \vartheta_1(z)$ and $\vartheta_2 = \vartheta_2(z)$ will not be calculated from the two differential equations, instead the variation in the difference between the temperature of the two fluids $\vartheta_1 - \vartheta_2$ will be determined. By subtracting (1.111) from (1.110) and dividing by $(\vartheta_1 - \vartheta_2)$ it follows that

$$\frac{\mathrm{d}(\vartheta_1 - \vartheta_2)}{\vartheta_1 - \vartheta_2} = (N_2 - N_1)\,\frac{\mathrm{d}z}{L} \ . \tag{1.112}$$

Integrating this differential equation between $z = 0$ and $z = L$ leads to

$$\ln\frac{(\vartheta_1 - \vartheta_2)_L}{(\vartheta_1 - \vartheta_2)_0} = \ln\frac{\vartheta_1'' - \vartheta_2'}{\vartheta_1' - \vartheta_2''} = N_2 - N_1 \ . \tag{1.113}$$

Now, we have

$$\frac{\vartheta_1'' - \vartheta_2'}{\vartheta_1' - \vartheta_2''} = \frac{\vartheta_1' - \vartheta_2' - (\vartheta_1' - \vartheta_1'')}{\vartheta_1' - \vartheta_2' - (\vartheta_2'' - \vartheta_2')} = \frac{1 - \varepsilon_1}{1 - \varepsilon_2} \ ,$$

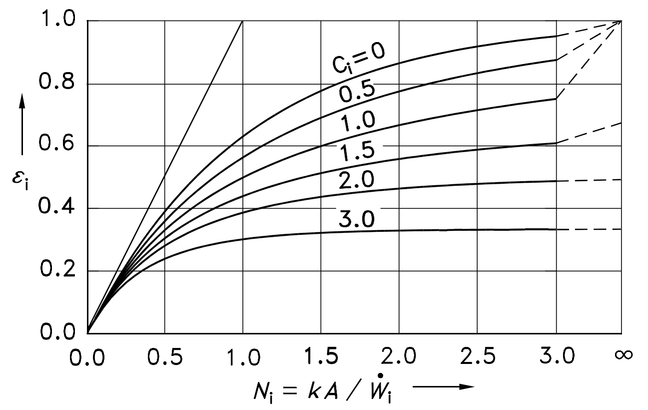

Fig. 1.32: Operating characteristic $\varepsilon_i = \varepsilon_i\,(N_i, C_i)$ for countercurrent flow from Tab. 1.4

which gives

$$\ln \frac{1-\varepsilon_1}{1-\varepsilon_2} = N_2 - N_1 \tag{1.114}$$

as the implicit form of the operating characteristic of a countercurrent heat exchanger. It is invariant with respect to an exchange of the indices 1 and 2. Using the ratios of C_1 and $C_2 = 1/C_1$ from (1.99) and (1.100), explicit equations are obtained,

$$\varepsilon_i = f(N_i, C_i) \quad \text{and} \quad N_i = f(\varepsilon_i, C_i) \quad , \qquad i = 1,2$$

which have the same form for both fluids. These explicit formulae for the operating characteristics are shown in Table 1.4. If the heat capacity flow rates are equal, $\dot{W}_1 = \dot{W}_2$, and because $C_1 = C_2 = 1$, it follows that

$$\varepsilon_1 = \varepsilon_2 = \varepsilon \quad \text{and} \quad N_1 = N_2 = N \ ,$$

and with a series development of the equations valid for $C_i \neq 1$ towards the limit of $C_i \to 1$, the simple relationships given in Table 1.4 are obtained.

Fig. 1.32 shows the operating characteristic $\varepsilon_i = f(N_i, C_i)$ as a function of N_i with C_i as a parameter. As expected the normalised temperature change ε_i grows monotonically with increasing N_i, and therefore increasing transfer capability kA. For $N_i \to \infty$ the limiting value is

$$\lim_{N_i \to \infty} \varepsilon_i = \begin{cases} 1 & \text{for} \quad C_i \leq 1 \\ 1/C_i & \text{for} \quad C_i > 1 \end{cases} \quad .$$

If $C_i \leq 1$, then ε_i takes on the character of an efficiency. The normalised temperature change of the fluid with the smaller heat capacity flow is known as the *efficiency* or *effectiveness of the heat exchanger*. With an enlargement of the heat transfer area A the temperature difference between the two fluids can be made as small as desired, but only at one end of the countercurrent exchanger. Only for

$\dot{W}_1 = \dot{W}_2$, which means $C_1 = C_2 = 1$, can an infinitely small temperature difference at at both ends, and therefore throughout the heat exchanger, be achieved by an enlargement of the surface area. The ideal case of reversible heat transfer between two fluids, often considered in thermodynamics, is thus only attainable when $\dot{W}_1 = \dot{W}_2$ in a heat exchanger with very high transfer capability.

As already mentioned in section 1.3.2, the function $\varepsilon_i = f(N_i, C_i)$ is used to calculate the outlet temperature and the transfer capability of a given heat exchanger. For the sizing of a heat exchanger for a required temperature change in the fluid, the other form of the operating characteristic, $N_i = N_i(\varepsilon_i, C_i)$, is used. This is also given in Table 1.4.

In a *cocurrent heat exchanger* the direction of flow is opposite to that in Fig. 1.31, cf. also Fig. 1.20b. In place of (1.108) the energy balance is

$$\mathrm{d}\dot{Q} = \dot{M}_2 c_{p2}\,\mathrm{d}\vartheta_2 = \dot{W}_2\,\mathrm{d}\vartheta_2 \ ,$$

which gives the relationship

$$\frac{\mathrm{d}(\vartheta_1 - \vartheta_2)}{\vartheta_1 - \vartheta_2} = -(N_1 + N_2)\,\frac{\mathrm{d}z}{L} \tag{1.115}$$

instead of (1.112). According to (1.114) the temperature difference between the two fluids in the direction of flow is always decreasing. Integration of (1.115) between $z = 0$ and $z = L$ yields

$$\ln\frac{\vartheta_1'' - \vartheta_2''}{\vartheta_1' - \vartheta_2'} = -(N_1 + N_2) \ ,$$

from which follows

$$\ln\left[1 - (\varepsilon_1 + \varepsilon_2)\right] = -(N_1 + N_2) = -\frac{\varepsilon_1 + \varepsilon_2}{\Theta} \tag{1.116}$$

as the implicit form of the operating characteristic. This can be solved for ε_i and N_i giving the functions noted in Table 1.4. For $N_i \to \infty$ the normalised temperature variation reaches the limiting value of

$$\lim_{N_i\to\infty} \varepsilon_i = \frac{1}{1 + C_i} \quad , \qquad i = 1, 2 \ .$$

With cocurrent flow the limiting value of $\varepsilon_i = 1$ is never reached except when $C_i = 0$, as will soon be explained.

The calculations for performance and sizing of a heat exchanger can also be carried out using a mean temperature difference Θ from (1.105) in section 1.3.2. In countercurrent flow, the difference $N_2 - N_1$ in (1.113) is replaced by Θ, ε_1 and ε_2 giving the expression $\Theta = \Theta(\varepsilon_1, \varepsilon_2)$ which appears in Table 1.4. Introducing

$$N_2 - N_1 = \frac{\varepsilon_2 - \varepsilon_1}{\Theta} = \frac{\vartheta_2'' - \vartheta_2' - (\vartheta_1' - \vartheta_1'')}{\Delta\vartheta_{\mathrm{m}}} = \frac{\vartheta_1'' - \vartheta_2' - (\vartheta_1' - \vartheta_2'')}{\Delta\vartheta_{\mathrm{m}}}$$

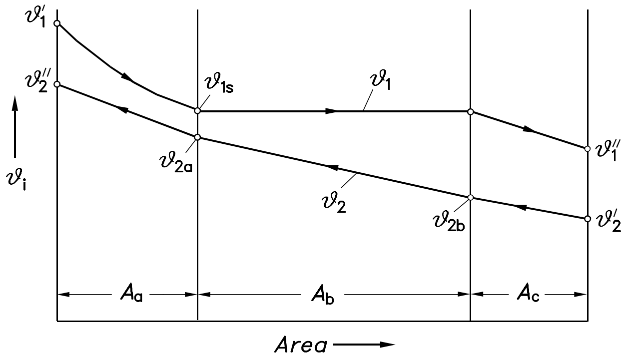

Fig. 1.33: Temperature in a condenser with cooling of superheated steam, condensation and subcooling of the condensate (fluid 1) by cooling water (fluid 2)

into (1.113) , gives

$$\Delta\vartheta_{\rm m} = \frac{\vartheta_1'' - \vartheta_2' - (\vartheta_1' - \vartheta_2'')}{\ln \dfrac{\vartheta_1'' - \vartheta_2'}{\vartheta_1' - \vartheta_2''}} \tag{1.117}$$

for the mean temperature difference in a countercurrent heat exchanger. It is the *logarithmic* mean of the temperature difference between the two fluids at both ends of the apparatus.

The expression, from (1.116), for the normalised mean temperature difference Θ, in cocurrent flow is given in Table 1.4. Putting in (1.117) the defining equations for ε_1 and ε_2 yields

$$\Delta\vartheta_{\rm m} = \frac{\vartheta_1' - \vartheta_2' - (\vartheta_1'' - \vartheta_2'')}{\ln \dfrac{\vartheta_1' - \vartheta_2'}{\vartheta_1'' - \vartheta_2''}} \ . \tag{1.118}$$

So $\Delta\vartheta_{\rm m}$ is also the logarithmic mean temperature difference at both ends of the heat exchanger in cocurrent flow.

We will now compare the two flow configurations. For $C_i = 0$ the normalised temperature variation in Table 1.4 is

$$\varepsilon_i = 1 - \exp(-N_i)$$

and the dimensionless transfer capability

$$N_i = -\ln(1 - \varepsilon_i)$$

both of which are independent of whether countercurrent or cocurrent flow is used. Therefore when one of the substances boils or condenses in the exchanger it is immaterial which flow configuration is chosen. However, if in a condenser, superheated steam is first cooled from ϑ_1' to the condensation temperature of ϑ_{1s}, then completely condensed, after which the condensate is cooled from ϑ_{1s} to ϑ_1'', more complex circumstances develop. In these cases it is not permissible to treat the equipment as simply one heat exchanger, using the equations that have already been defined, where only the inlet and outlet temperatures ϑ_i' and

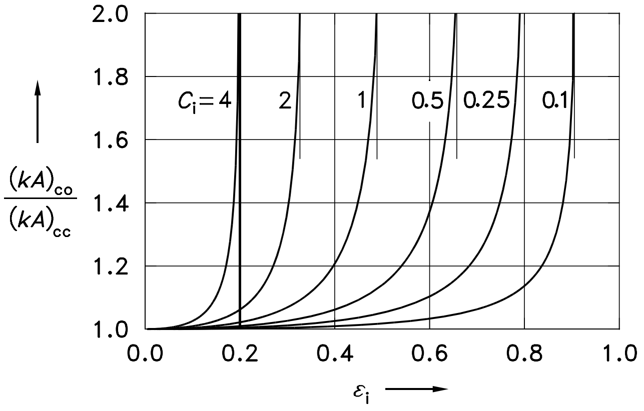

Fig. 1.34: Ratio $(kA)_{\mathrm{co}} / (kA)_{\mathrm{cc}} = N_i^{\mathrm{co}}/N_i^{\mathrm{cc}}$ of the transfer capabilities in cocurrent (index co) and countercurrent (index cc) flows as a function of ε_i and C_i

ϑ_i'' $(i = 1, 2)$ are important, cf. Fig. 1.33. The values for the heat capacity flow rate $\dot{W}_1$ change significantly: During the cooling of the steam and the condensate $\dot{W}_1$ has a finite value, whereas in the process of condensation $\dot{W}_1$ is infinite. The exchanger has to be imaginarily split, and then be treated as three units in series. Energy balances provide the two unknown temperatures, $\vartheta_{2\mathrm{a}}$ between the cooling and condensation section, and $\vartheta_{2\mathrm{b}}$, between the condensation and sub-cooling part. These in turn yield the dimensionless temperature differences $\varepsilon_{i\mathrm{a}}$, $\varepsilon_{i\mathrm{b}}$ and $\varepsilon_{i\mathrm{c}}$ for the three sections cooler a, condenser b and sub-cooler c $(i = 1, 2)$. The dimensionless transfer capabilities $N_{i\mathrm{a}}$, $N_{i\mathrm{b}}$ and $N_{i\mathrm{c}}$ of the three equipment sections can then be calculated according to the relationships in Table 1.4. From N_{ij} the values for $(kA)_j$ can be found. Then using the relevant overall heat transfer coefficients k_j, we obtain the areas of the three sections A_j $(j = \mathrm{a, b, c})$, which together make up the total transfer area of the exchanger.

For $C_i > 0$ the countercurrent configuration is always superior to the cocurrent. A disadvantage of the cocurrent flow exists in that not all heat transfer tasks can be solved in such a system. A given temperature change ε_i is only realisable if the argument of the logarithmic term in

$$N_i^{\mathrm{co}} = -\frac{1}{1 + C_i} \ln\left[1 - \varepsilon_i(1 + C_i)\right]$$

is positive. This is only the case for

$$\varepsilon_i < \frac{1}{1 + C_i} \quad . \tag{1.119}$$

Larger normalised temperature changes cannot be achieved in cocurrent heat exchangers even in those with very large values for the transfer capability kA. In countercurrent exchangers this limitation does not exist. All values for ε_i are basically attainable and therefore all required heat loads can be transferred as long as the area available for heat transfer is made large enough.

A further disadvantage of cocurrent flow is that a higher transfer capability kA is required to fulfill the same task (same ε_i and C_i) when compared with a countercurrent system. This is shown in Fig. 1.34 in which the ratio

$$(kA)_{\text{co}}/(kA)_{\text{cc}} = N_i^{\text{co}}/N_i^{\text{cc}}$$

based on the equations in Table 1.4 is represented. This ratio grows sharply when ε_i approaches the limiting value according to (1.119). Even when a cocurrent exchanger is capable of fulfilling the requirements of the task, the countercurrent exchanger will be chosen as its dimensions are smaller. Only in a combination of small enough values of C_i and ε_i the necessary increase in the area of a cocurrent exchanger is kept within narrow limits.

Example 1.4: Ammonia, at a pressure of 1.40 MPa, is to be cooled in a countercurrent heat exchanger from $\vartheta_1' = 150.0\ °\text{C}$ to the saturation temperature $\vartheta_{1\text{s}}' = 36.3\ °\text{C}$, and then completely condensed. Its mass flow rate is $\dot{M}_1 = 0.200\,\text{kg/s}$. Specific enthalpies of $h(\vartheta_1') = 1797.1\,\text{kJ/kg}$, $h^{\text{g}}(\vartheta_{1\text{s}}) = 1488.8\,\text{kJ/kg}$, and $h^{\text{fl}}(\vartheta_{1\text{s}}) = 372.2\,\text{kJ/kg}$ are taken from the property tables for ammonia, [1.13]. Cooling water with a temperature of $\vartheta_2' = 12.0\ °\text{C}$ is available, and this can be heated to $\vartheta_2'' = 28.5\ °\text{C}$. Its mean specific heat capacity is $\overline{c}_{p2} = 4.184\,\text{kJ/kgK}$. The required transfer capabilities for the cooling $(kA)_{\text{cooling}}$ and $(kA)_{\text{cond}}$ for the condensation of the ammonia have to be determined.
At first the heat flow transferred $\dot{Q}$, and the required mass flow rate $\dot{M}_2$ of water have to be found. The heat flow removed from the ammonia is

$$\dot{Q} = \dot{M}_1 \left[h(\vartheta_1') - h^{\text{fl}}(\vartheta_{1\text{s}})\right] = 0.200\,\frac{\text{kg}}{\text{s}}\,(1797.1 - 372.2)\,\frac{\text{kJ}}{\text{kg}} = 285.0\,\text{kW} \quad .$$

From that the mass flow rate of water is found to be

$$\dot{M}_2 = \frac{\dot{Q}}{\overline{c}_{p2}\,(\vartheta_2'' - \vartheta_1')} = \frac{285.0\,\text{kW}}{4.184\,(\text{kJ/kgK})\,(28.5 - 12.0)\ \text{K}} = 4.128\,\frac{\text{kg}}{\text{s}} \quad .$$

The temperature $\vartheta_{2\text{a}}$ of the cooling water in the cross section between the cooling and condensation sections, cf. Fig. 1.35, is required to calculate the transfer capability.

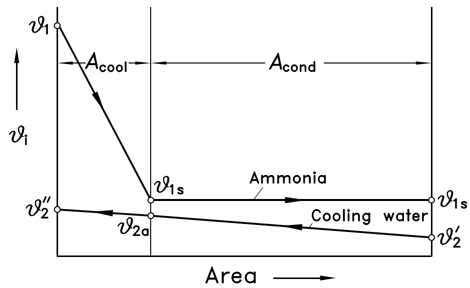

Fig. 1.35: Temperatures of ammonia and cooling water in a countercurrent heat exchanger (schematic)

From the energy balance for the condensor section

$$\dot{M}_2\overline{c}_{p2}\,(\vartheta_{\text{a}} - \vartheta_2') = \dot{M}_1\left[h^{\text{g}}(\vartheta_{1\text{s}}) - h^{\text{fl}}(\vartheta_{1\text{s}})\right] \quad ,$$

it follows that

$$\vartheta_{2\text{a}} = \vartheta_2' + \frac{\dot{M}_1}{\dot{M}_2\overline{c}_{p2}}\left[h^{\text{g}}(\vartheta_{1\text{s}}) - h^{\text{fl}}(\vartheta_{1\text{s}})\right] = 24.9\ °\text{C} \quad .$$

The transfer capability for the ammonia cooling section, using Table 1.4, is

$$\frac{(kA)_{\text{cooling}}}{\dot{W}_1} = N_1 = \frac{1}{1-C_1}\ln\frac{1-C_1\varepsilon_1}{1-\varepsilon_1} \quad . \tag{1.120}$$

The ratio of the heat capacity flow rates is found with

$$\dot{W}_1 = \dot{M}_1\overline{c}_{p1} = \dot{M}_1\frac{h(\vartheta_1') - h^{\text{g}}(\vartheta_{\text{s}})}{\vartheta_1' - \vartheta_{\text{s}}} = 0.200\,\frac{\text{kg}}{\text{s}}\,\frac{1797.1 - 1488.8}{150.0 - 36.3}\,\frac{\text{kJ}}{\text{kgK}} = 0.5423\,\frac{\text{kW}}{\text{K}}$$

and with $\dot{W}_2 = \dot{M}_2\overline{c}_{p2} = 17.272\,\text{kW/K}$ giving $C_1 = 0.0314$. The dimensionless temperature variation of ammonia is

$$\varepsilon_1 = \frac{\vartheta_1' - \vartheta_{\text{s}}}{\vartheta_1' - \vartheta_{2\text{a}}} = \frac{150.0 - 36.3}{150.0 - 24.9} = 0.9089.$$

Then (1.120) yields $N_1 = 2.443$ and finally

$$(kA)_{\text{cooling}} = N_1\dot{W}_1 = 1.325\,\text{kW/K} \quad .$$

For the *condensation section* of the heat exchanger $\varepsilon_1 = 0$, and because $\dot{W}_1 \to \infty$ this means $C_2 = \dot{W}_2/\dot{W}_1 = 0$. From Table 1.4 it follows that

$$(kA)_{\text{cond}}/\dot{W}_2 = N_2 = -\ln(1-\varepsilon_2) \quad .$$

With the normalised temperature change of the cooling water

$$\varepsilon_2 = \frac{\vartheta_{2\text{a}} - \vartheta_2'}{\vartheta_{1\text{s}} - \vartheta_2'} = \frac{24.9 - 12.0}{36.3 - 12.0} = 0.5309 \quad ,$$

yielding $N_2 = 0.7569$, which then gives

$$(kA)_{\text{cond}} = N_2\dot{W}_2 = 13.07\,\text{kW/K} \quad .$$

In order to find the required area $A = A_{\text{cooling}} + A_{\text{cond}}$ for the countercurrent exchanger, from the values for $(kA)_{\text{cooling}}$ and $(kA)_{\text{cond}}$, the overall heat transfer coefficients for each part must be calculated. They will be different as the resistance to heat transfer in the cooling section is greatest on the gaseous ammonia side, whereas in the condensation section the greatest resistance to heat transfer is experienced on the cooling water side. The calculations for the overall heat transfer coefficients will not be done here as the design of the heat exchanger and the flow conditions have to be known for this purpose.

1.3.4 Crossflow heat exchangers

Before discussing pure crossflow as shown in Fig. 1.23, the operating characteristic for the simple case of crossflow where only the fluid on one side is laterally mixed will be calculated. In this flow configuration the temperature of one of the two fluids is only dependent on *one* position coordinate, e.g. x, while the temperature of the other fluid changes with both x and y. In Fig. 1.36 the laterally mixed fluid is indicated by the index 1. Its temperature ϑ_1 changes only in the direction of flow, $\vartheta_1 = \vartheta_1(x)$. Ideal lateral mixing is assumed so that $\vartheta_1$does not vary with y. This assumption is closely met when fluid 1 flows through a single row of tubes

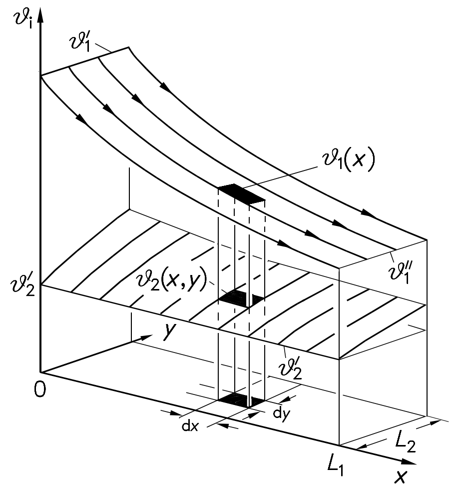

Fig. 1.36: Temperature variations in a one side laterally mixed crossflow. $\vartheta_1 = \vartheta_1(x)$ temperature of the laterally mixed fluid, $\vartheta_2 = \vartheta_2(x,y)$ temperature of the other fluid

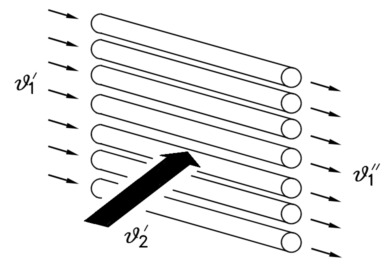

Fig. 1.37: Crossflow with one tube row as a realisation of the one side laterally mixed crossflow

and fluid 2 flows perpendicular to them, Fig. 1.37. This crossflow with a single row of tubes corresponds to one side laterally mixed crossflow. The mixed fluid 1 in the tubes does not have to be the fluid with the higher temperature, as was assumed before.

To determine the temperatures $\vartheta_1 = \vartheta_1(x)$ and $\vartheta_2 = \vartheta_2(x,y)$ of both fluids, the surface element, $\mathrm{d}A = \mathrm{d}x\,\mathrm{d}y$ picked out in Fig. 1.36 will be considered. The heat flow transferred from fluid 1 to fluid 2 is given as

$$\mathrm{d}\dot{Q} = [\vartheta_1(x) - \vartheta_2(x,y)]\,k\,\mathrm{d}x\,\mathrm{d}y \quad .$$

The total heat transfer area is $A = L_1 L_2$, see Fig. 1.36. With the dimensionless coordinates

$$x^+ := x/L_1 \quad \text{and} \quad y^+ := y/L_2 \tag{1.121}$$

it follows that

$$\mathrm{d}\dot{Q} = \left[\vartheta_1(x^+) - \vartheta_2(x^+, y^+)\right] kA\,\mathrm{d}x^+\,\mathrm{d}y^+ \quad . \tag{1.122}$$

A second relationship for $\mathrm{d}\dot{Q}$ is yielded from the application of the first law on fluid 2, which flows over the surface element $\mathrm{d}A$. Its mass flow rate is

$$\mathrm{d}\dot{M}_2 = \dot{M}_2\,\mathrm{d}x/L_1 = \dot{M}_2\,\mathrm{d}x^+ \quad ,$$

which gives

$$\mathrm{d}\dot{Q} = \dot{M}_2\,\mathrm{d}x^+ c_{p2}\left(\vartheta_2 + \frac{\partial \vartheta_2}{\partial y^+}\,\mathrm{d}y^+ + \ldots - \vartheta_2\right) = \dot{M}_2 c_{p2}\left(\frac{\partial \vartheta_2}{\partial y^+}\,\mathrm{d}y^+ + \ldots\right)\mathrm{d}x^+$$

or

$$\mathrm{d}\dot{Q} = \dot{W}_2 \frac{\partial \vartheta_2}{\partial y^+}\,\mathrm{d}x^+\,\mathrm{d}y^+ \quad . \tag{1.123}$$

The differential equation

$$\frac{\partial \vartheta_2}{\partial y^+} = N_2(\vartheta_1 - \vartheta_2) \tag{1.124}$$