# Dynamics of Multibody Systems

*Third Edition*

*Dynamics of Multibody Systems*, Third Edition, introduces multibody dynamics, with an emphasis on flexible body dynamics. Many common mechanisms such as automobiles, space structures, robots, and micromachines have mechanical and structural systems that consist of interconnected rigid and deformable components. The dynamics of these large-scale, multibody systems are highly nonlinear, presenting complex problems that in most cases can only be solved with computer-based techniques. The book begins with a review of the basic ideas of kinematics and the dynamics of rigid and deformable bodies before moving on to more advanced topics and computer implementation. This revised third edition now includes important new developments relating to the problem of large deformations and numerical algorithms as applied to flexible multibody systems. The book's wealth of examples and practical applications will be useful to graduate students, researchers, and practicing engineers working on a wide variety of flexible multibody systems.

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# DYNAMICS OF MULTIBODY SYSTEMS

## *Third Edition*

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*To my father*

*and to*

*the memory of my mother*

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# Contents

# Preface

The methods for the nonlinear analysis of physical and mechanical systems developed for use on modern digital computers provide means for accurate analysis of large-scale systems under dynamic loading conditions. These methods are based on the concept of replacing the actual system by an equivalent model made up from discrete bodies having known elastic and inertia properties. The actual systems, in fact, form multibody systems consisting of interconnected rigid and deformable bodies, each of which may undergo large translational and rotational displacements. Examples of physical and mechanical systems that can be modeled as multibody systems are machines, mechanisms, vehicles, robotic manipulators, and space structures. Clearly, these systems consist of a set of interconnected bodies which may be rigid or deformable. Furthermore, the bodies may undergo large relative translational and rotational displacements. The dynamic equations that govern the motion of these systems are highly nonlinear which in most cases cannot be solved analytically in a closed form. One must resort to the numerical solution of the resulting dynamic equations.

The aim of this text, which is based on lectures that I have given during the past several years, is to provide an introduction to the subject of multibody mechanics in a form suitable for senior undergraduate and graduate students. The initial notes for the text were developed for two first-year graduate courses introduced and offered at the University of Illinois at Chicago. These courses were developed to emphasize both the general methodology of the nonlinear dynamic analysis of multibody systems and its actual implementation on the high-speed digital computer. This was prompted by the necessity to deal with complex problems arising in modern engineering and science. In this text, an attempt has been made to provide the rational development of the methods from their foundations and develop the techniques in clearly understandable stages. By understanding the basis of each step, readers can apply the method to their own problems.

The material covered in this text comprises an introductory chapter on the subjects of kinematics and dynamics of rigid and deformable bodies. In this chapter some

background materials and a few fundamental ideas are presented. In Chapter 2, the kinematics of the body reference is discussed and the transformation matrices that define the orientation of this body reference are developed. Alternate forms of the transformation matrix are presented. The material presented in this chapter is essential for understanding the dynamic motion of both rigid and deformable bodies. Analytical techniques for deriving the system differential and algebraic equations of motion of a multibody system consisting of rigid bodies are discussed in Chapter 3. In Chapter 4, an introduction to the theory of elasticity is presented. The material covered in this chapter is essential for understanding the dynamics of deformable bodies that undergo large translational and rotational displacements. In Chapter 5, the equations of motion of deformable multibody systems in which the reference motion and elastic deformation are coupled are derived using classical approximation methods. In Chapters 6 and 7, two finite element formulations are presented. Both formulations lead to exact modeling of the rigid body inertia and lead to zero strains under an arbitrary rigid body motion. The first formulation discussed in Chapter 6, which is based on the concept of the intermediate element coordinate system, uses the definition of the coordinates used in the conventional finite element method. A conceptually different finite element formulation that can be used in the large deformation analysis of multibody systems is presented in Chapter 7. In this chapter, the absolute nodal coordinate formulation in which no infinitesimal or finite rotations are used as element coordinates is introduced.

I am grateful to many teachers, colleagues, and students who have contributed to my education in this field. I owe a particular debt of gratitude to Dr. R.A. Wehage and Dr. M.M. Nigm for their advice, encouragement, and assistance at various stages of my educational career. Their work in the areas of computational mechanics and vibration theory stimulated my early interest in the subject of nonlinear dynamics. Several chapters of this book have been read, corrected, and improved by many of my graduate students. I would like to acknowledge the collaboration with my students Drs. Om Agrawal, E. Mokhtar Bakr, Ipek Basdogan, Michael Brown, Bilin Chang, Che-Wei Chang, Koroosh Changizi, Da-Chih Chen, Jui-Sheng Chen, Jin-Hwan Choi, Hanaa El-Absy, Marian Gofron, Wei-Hsin Gau, Wei-Cheng Hsu, Kuo-Hsing Hwang, Yunn-Lin Hwang, Yehia Khulief, John Kremer, Haichiang Lee, Jalil Rismantab-Sany, Mohammad Sarwar, Marcello Berzeri, Marcello Campanelli, Andrew Christensen, Hussien Hussien, Refaat Yakoub, and Hiroyuki Sugiyama. Their work contributed significantly to the development of the material presented in this book. Special thanks are due to Ms. Denise Burt for the excellent job in typing most of the manuscript. Finally, I thank my family for their patience and encouragement during the time of preparation of this text.

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